Trigonometria

Gradi	**0**	30	45	60	**90**	120	135	150	**180**	210	225	240	**270**	300	315	330	**360**
Radianti	**0**	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
Sin	**0**	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	**1**	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	**0**	-1/2	$-\sqrt{2}/2$	$-\sqrt{3}/2$	**-1**	$-\sqrt{3}/2$	$-\sqrt{2}/2$	-1/2	**0**
Cos	**1**	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	**0**	-1/2	$-\sqrt{2}/2$	$-\sqrt{3}/2$	**-1**	$-\sqrt{3}/2$	$-\sqrt{2}/2$	-1/2	**0**	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	**1**
Tan	**0**	$\sqrt{3}/3$	1	$\sqrt{3}$	**-**	$-\sqrt{3}$	-1	$-\sqrt{3}/3$	**0**	$\sqrt{3}/3$	1	$\sqrt{3}$	**-**	$-\sqrt{3}$	-1	$-\sqrt{3}/3$	**0**

Identità trigonometriche:

$\mathrm{cosec}(\alpha) = 1/\mathrm{sen}(\alpha)$
$\sec(\alpha) = 1/\cos(\alpha)$
$\cot(\alpha) = 1/\mathrm{tg}(\alpha)$
$\mathrm{tg}(\alpha) = \mathrm{sen}(\alpha)/\cos(\alpha)$
$\mathrm{cotg}(\alpha) = \cos(\alpha)/\mathrm{sen}(\alpha)$

Funzioni Iperboliche:

$\mathrm{sh}(\alpha) = ½ (e^{\alpha} - e^{-\alpha})$
$\mathrm{ch}(\alpha) = ½ (e^{\alpha} + e^{-\alpha})$
$\mathrm{th}(\alpha) = \mathrm{sh}(\alpha) / \mathrm{ch}(\alpha)$

Relazioni tra angoli:

$\cos(\pi/2-\alpha) = \mathrm{sen}(\alpha)$
$\mathrm{sen}(\pi/2-\alpha) = \cos(\alpha)$
$\mathrm{tg}(\pi/2-\alpha) = \mathrm{cotg}(\alpha)$
$\mathrm{sen}(-\alpha) = -\,\mathrm{sen}(\alpha)$
$\cos(-\alpha) = \cos(\alpha)$
$\mathrm{tg}(-\alpha) = -\mathrm{tg}(\alpha)$
$\cot(-\alpha) = -\mathrm{cotg}(\alpha)$
$\sec(-\alpha) = \sec(\alpha)$
$\csc(-\alpha) = -\csc(\alpha)$

Relazioni di Pitagora:

$\cos^2(\alpha)+\mathrm{sen}^2(\alpha) = 1$
$\mathrm{ch}^2(\alpha)-\mathrm{sh}^2(\alpha) = 1$
$1+\mathrm{tg}^2(\alpha) = \sec^2(\alpha)$
$1+\mathrm{cotg}^2(\alpha) = \mathrm{cosec}^2(\alpha)$

Formule di Werner:

$\mathrm{sen}(\alpha)\cos(\beta) = ½ [\mathrm{sen}(\alpha+\beta)+\mathrm{sen}(\alpha-\beta)]$
$\cos(\alpha)\mathrm{sen}(\beta) = ½ [\mathrm{sen}(\alpha+\beta)-\mathrm{sen}(\alpha-\beta)]$
$\cos(\alpha)\cos(\beta) = ½ [\cos(\alpha+\beta)+\cos(\alpha-\beta)]$
$\mathrm{sen}(\alpha)\mathrm{sen}(\beta) = ½ [\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

Formule di duplicazione:

$\mathrm{sen}(2\alpha) = 2\mathrm{sen}(\alpha)\cos(\alpha)$
$\cos(2\alpha) = \cos^2(\alpha)-\mathrm{sen}^2(\alpha)$
$\mathrm{tg}(2\alpha) = 2\mathrm{tg}(\alpha)/(1-\mathrm{tg}^2(\alpha))$

Formule di bisezione:

$\cos^2(\alpha/2) = ½ (1+\cos(\alpha))$
$\mathrm{sen}^2(\alpha/2) = ½ (1-\cos(\alpha))$
$\mathrm{tg}^2(\alpha/2) = (1-\cos(\alpha))/(1+\cos(\alpha))$

Formule di addizione e sottrazione:

$\mathrm{sen}(\alpha\pm\beta) = \mathrm{sen}(\alpha)\cos(\beta)\pm\mathrm{sen}(\beta)\cos(\alpha)$
$\cos(\alpha\pm\beta) = \cos(\alpha)\cos(\beta)\square\,\mathrm{sen}(\alpha)\mathrm{sen}(\beta)$
$\mathrm{tg}(\alpha\pm\beta) = [\mathrm{tg}(\alpha)\pm\mathrm{tg}(\beta)]/[1\square\,\mathrm{tg}(\alpha)\mathrm{tg}(\beta)]$

Formule di prostaferisi:

$\mathrm{sen}(\alpha) + \mathrm{sen}(\beta) = 2\mathrm{sen}(\ ½\ (\alpha+\beta))\cos(\ ½\ (\alpha-\beta))$
$\mathrm{sen}(\alpha) - \mathrm{sen}(\beta) = 2\cos(\ ½\ (\alpha+\beta))\mathrm{sen}(\ ½\ (\alpha-\beta))$
$\cos(\alpha) + \cos(\beta) = 2\cos(\ ½\ (\alpha+\beta))\cos(\ ½\ (\alpha-\beta))$
$\cos(\alpha) - \cos(\beta) = 2\mathrm{sen}(\ ½\ (\alpha+\beta))\mathrm{sen}(\ ½\ (\alpha-\beta))$
$\mathrm{sen}(\alpha-\beta)\cos(\alpha+\beta) = \mathrm{sen}(\beta)\cos(\beta)-\cos(\alpha)\mathrm{sen}(\alpha)$
$\mathrm{sen}(\alpha+\beta)\cos(\alpha-\beta) = \mathrm{sen}(\beta)\cos(\beta)+\cos(\alpha)\mathrm{sen}(\alpha)$

Formule di Eulero:

$e^{i\alpha} = \cos(\alpha) + i\,\mathrm{sen}(\alpha)$
$e^{-i\alpha} = \cos(\alpha) - i\,\mathrm{sen}(\alpha)$
$e^{i(\alpha+2k\pi)} = e^{i\alpha}$; (k intero relativo)
$\cos(\alpha) = (e^{i\alpha} + e^{-i\alpha})/2$
$\mathrm{sen}(\alpha) = (e^{i\alpha} - e^{-i\alpha})/(2i)$

Diseguaglianze tipiche:

$\mathrm{sen}(\alpha)>0$ per $2k\pi < \alpha < (2k+1)\pi$
$\cos(\alpha)>0$ per $2k\pi-\pi/2 < \alpha < 2k\pi+\pi/2$

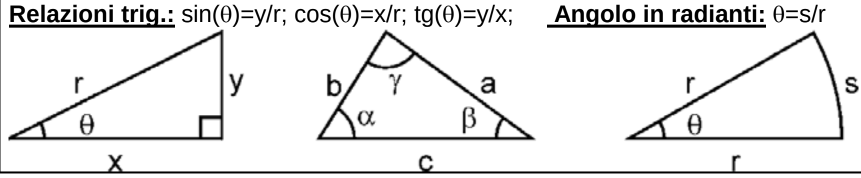
Teorema dei seni:

$\mathrm{sen}(\alpha)/a = \mathrm{sen}(\beta)/b = \mathrm{sen}(\gamma)/c$

Teorema del coseno:

$c^2 = a^2+b^2-2ab\cos(\gamma)$

Relazioni trig.: $\sin(\theta)=y/r$; $\cos(\theta)=x/r$; $\mathrm{tg}(\theta)=y/x$; **Angolo in radianti:** $\theta=s/r$



Utilità (Algebra e Geometria):

Potenze:

$a^m a^n = a^{n+m}$
$a^m/a^n = a^{m-n}$
$(a^m)^n = a^{mn}$
$(ab)^n = a^n b^n$
$(a/b)^n = a^n/b^n$

Logaritmi: $a^b = c \Leftrightarrow \lg_a(c) = b$

$\lg_a(a) = 1$
$\lg_b(a) = 1/\lg_a(b)$
$\lg_b(MN) = \lg_b(M) + \lg_b(N)$
$\lg_b(M/N) = \lg_b(M) - \lg_b(N) = -\lg_b(N/M)$
$\lg_b(N^y) = y\,\lg_b(N)$

Radicali:

${}^n\sqrt{a^m} = ({}^n\sqrt{a})^m = a^{m/n}$
${}^n\sqrt{{}^m\sqrt{a}} = {}^{nm}\sqrt{a}$
${}^n\sqrt{a/b} = {}^n\sqrt{a} / {}^n\sqrt{b}$
$\sqrt{a\pm\sqrt{b}} = \sqrt{½ (a+\sqrt{a^2+b})} \pm \sqrt{½ (a-\sqrt{a^2-b})}$

Geometria:

Parallelogramma $A=bh$
Triangolo $A = ½\ bh$
Cerchio $A = \pi r^2$; $C = 2\pi r$
Trapezio $A = ½\ (a+b)h$
Paralleloepipedo rettangolare
$S = 2(lw+wh+lh)$; $V = lwh$
Sfera $S = 4\pi r^2$; $V = (4/3)\ \pi r^3$
Cilindro retto $S = 2\pi r(r+h)$;
$V = \pi r^2 h$
Cono retto $S = \pi r(r+\sqrt{h^2+r^2})$;
$V = 1/3\ \pi r^2\ h$

Prodotti notevoli:

$(a\pm b)^2 = a^2\pm 2ab+b^2$
$(a\pm b)^3 = a^3\pm 3a^2b+3ab^2\text{-}b^3$

Scomposizione in fattori:

$a^3+b^3 = (a+b)(a^2-ab+b^2)$
$a^3-b^3 = (a-b)(a^2+ab+b^2)$

Equazioni di secondo grado:

Se $ax^2+bx+c = 0$ per $a\neq 0$ allora
$x = (1/2a)(-b\pm\sqrt{b^2-4ac})$

Proprietà dei vettori: $\mathbf{x} = \sum_{i,1,n} x_i\,\mathbf{e}_i$

$\mathbf{x}+\mathbf{y} = \mathbf{y}+\mathbf{x}$ $\quad \mathbf{x} + \mathbf{y} = (x_i + y_i)\mathbf{e}_i$
$\mathbf{x}+0 = \mathbf{x}$
$\mathbf{x}+(-\mathbf{x}) = 0$
$(\mathbf{x}+\mathbf{y})+\mathbf{z} = \mathbf{x}+(\mathbf{y}+\mathbf{z})$
$(a+b)\mathbf{x} = a\mathbf{x}+b\mathbf{x}$
$a(\mathbf{x}+\mathbf{y}) = a\mathbf{x}+a\mathbf{y}$
$(ab)\mathbf{x} = a(b\mathbf{x})$
$1\cdot\mathbf{x} = \mathbf{x}$
$\mathbf{x} = (\mathbf{x}\cdot\mathbf{n})\mathbf{n} + \mathbf{n}\wedge(\mathbf{x}\wedge\mathbf{n})$

Binomio di Newton:

$(a\pm b)^n = \sum_{k,0,n} (\pm 1)^k\ ({}^n{}_k)\ a^{(n-k)}\ b^k$

Distanza:

posto $\mathbf{x}$ = P-O; $\mathbf{y}$ = Q-O;
$|P-Q| = \sqrt{[(\mathbf{x}-\mathbf{y})\cdot(\mathbf{x}-\mathbf{y})]}$

Altre proprietà:

$(\mathbf{x}\wedge\mathbf{y})\wedge\mathbf{z} = (\mathbf{x}\cdot\mathbf{z})\mathbf{y} - (\mathbf{y}\cdot\mathbf{z})\mathbf{x}$
$\mathbf{x}\wedge(\mathbf{y}\wedge\mathbf{z}) = (\mathbf{x}\cdot\mathbf{z})\mathbf{y} - (\mathbf{x}\cdot\mathbf{y})z$

Prodotto vettoriale:

$|\mathbf{x}\wedge\mathbf{y}| = |\mathbf{x}||\mathbf{y}||\sin(\theta)|$
$|\mathbf{x}\wedge\mathbf{y}|^2 = |\mathbf{x}|^2|\mathbf{y}|^2-(\mathbf{x}\cdot\mathbf{y})^2$
$\mathbf{x}\wedge\mathbf{y} = -\mathbf{y}\wedge\mathbf{x}$
$\mathbf{x}\wedge(a\mathbf{y}+b\mathbf{z}) = a\mathbf{x}\wedge\mathbf{y} + b\mathbf{x}\wedge\mathbf{z}$
$\mathbf{x}\wedge\mathbf{y} = 0$ per $\mathbf{x},\mathbf{y}$ paralleli

$$\mathbf{x}\wedge\mathbf{y} = \det\begin{vmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{vmatrix}$$

Prodotto misto:

$\mathbf{x}\cdot\mathbf{y}\wedge\mathbf{z} = \mathbf{y}\cdot\mathbf{z}\wedge\mathbf{x} = \mathbf{z}\cdot\mathbf{x}\wedge\mathbf{y}$
$\mathbf{x}\wedge\mathbf{y}\cdot\mathbf{z} = -(\mathbf{z}\wedge\mathbf{x}\cdot\mathbf{y})$

Prodotto scalare:

$\mathbf{x}\cdot\mathbf{y} = |\mathbf{x}||\mathbf{y}|\cos(\theta)$
$|\mathbf{x}| = \sqrt{(\mathbf{x}\cdot\mathbf{x})}$
$\mathbf{x}\cdot\mathbf{y} = \mathbf{y}\cdot\mathbf{x}$
$(a\mathbf{x}+b\mathbf{y})\cdot\mathbf{z} = a\mathbf{x}\cdot\mathbf{z} + b\mathbf{y}\cdot\mathbf{z}$
$\mathbf{x}\cdot\mathbf{x} \geq 0$
$\mathbf{x}\cdot\mathbf{y} = 0$ per $\mathbf{x},\mathbf{y}$ perpendicolari

Calcolo combinatorio:

Disposizioni con ripetizione: $R_{n,k} = n^k$
Disposizioni semplici: $D_{n,k} = n(n-1)(n-2)...(n-k+1)$
Permutazioni: $D_{n,n} = n!$
Combinazioni: $C_{n,k} = D_{n,k} / k! = n! / [k!(n-k)!] = ({}^n{}_k)$

Determinante 2X2: $\det(A) = |A|$

$$\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Determinante n x n:

Si sommano gli elementi di una riga o di una colonna per i determinanti delle sottomatrici ottenute eliminando la riga e la colonna dell'elemento considerato. (Tecnica ricorsiva).

Derivate ed Integrali

Derivazione per funzioni:	**Vettori:**	**Integrazione:**
$D[f(x)^n] = n[f(x)]^{n-1}f'(x)$	$d\mathbf{x}(u)/du = \Sigma_{i,1,3}(dx_i/du)e_i$	$\int f(x)\,dx = F(x) + C$
$D[f(x)+g(x)] = f'(x)+g'(x)$	$d(\mathbf{x}+\mathbf{y})/du = d\mathbf{x}/du + d\mathbf{y}/du$	$\int_{a,b} f(x)\,dx = [F(x)]_{a,b} = F(b) - F(a)$
$D[f(x)g(x)] = f'(x)g(x)+f(x)g'(x)$	$d(a\mathbf{x})/du = (da/du)\mathbf{x} + (d\mathbf{x}/du)a$	$\int_{a,b} f(x)\,dx = \int_{a,c} f(x)\,dx + \int_{c,b} f(x)\,dx$; per $a<c<b$
$D[f(x)/g(x)] = [1/g(x)^2][f'(x)g(x)-f(x)g'(x)]$	$d(\mathbf{x}\cdot\mathbf{y})/du = (d\mathbf{x}/du)\mathbf{y} + (d\mathbf{y}/du)\mathbf{x}$	$\int k\,f(x)\,dx = k\int f(x)\,dx$
$D[1/f(x)] = -f'(x)/f(x)^2$	$d(\mathbf{x}\wedge\mathbf{y})/du = (d\mathbf{x}/du)\wedge\mathbf{y}+\mathbf{x}\wedge(d\mathbf{y}/du)$	$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$
$D[f(g(x))] = f'(g(x))g'(x)$	$d\mathbf{x}(u(w))/du = (d\mathbf{x}/du)\cdot(du/dw)$	$\int_{a,b} f(x)g(x)\,dx = [G(x)f(x)]_{a,b} - \int_{a,b} G(x)f'(x)\,dx$
$D[a^{f(x)}] = a^{f(x)}\log(a)f'(x)$	$\int_{U0,U1} \mathbf{x}(u)\,du = \mathbf{e}_i\int_{U0,U1} x_i(u)\,du$	$\int f(\varphi(x))\varphi'(x)\,dx = F(\varphi(x)) + C$
$D[f(x)^{g(x)}] = D[e^{g(x)\ln(f(x))}]$	**Teorema dei residui:**	$\int_{\varphi(\alpha),\varphi(\beta)} f(x)\,dx = \int_{\alpha,\beta} f(\varphi(t))\varphi(t)\,dt$
$D[f^{-1}(y)] = 1/f'(x_0)$	$\int_\sigma f(z)\,dz = 2\pi i\,\Sigma_{k,1,n}\,\Re(z_k)$	$\int f'(x)/f(x)\,dx = \lg\lvert f(x)\rvert + C$
$D[f(x)g(x)]^{(n)} = \Sigma_{k,0,n}\binom{n}{k} f^{(k)}(x)\,g^{(n-k)}(x)$	con z_k singolarità o poli contenuti in σ.	$\int f'(x)/(f(x))^n\,dx = f(x)^{1-n}/(1-n)$

INTEGRALI ⇒	⇐ DERIVATE
1	X
Knx^{n-1}	Kx^n
x^n	$X^{n+1}/n+1$
$\ln(ax)$	$X\ln(ax) - x$
$1/x$	$\mathrm{Ln}(x)$
$1/(x\ln(a))$	$\mathrm{Log}_a(x)$
e^x	E^x
$ba^{bx}\ln(a)$	A^{bx}
ba^{bx}	$A^{bx}/\ln(a)$
$x\,a^{bx}$	$A^{bx}\,[x/(b\ln(a)) - 1/(b^2\ln(a)^2)]$
$1/(a - ba^{bx})$	$x/a - [\ln(ba^{bx} - a)]\,/\,[a\,b\ln(a)]$
$\sin(ax)$	$-(1/a)\cos(ax)$
$\cos(ax)$	$(1/a)\sin(ax)$
$1/\cos^2(ax)$	$[\mathrm{tg}(ax)]/a$
$1/(1+\cos(x))$	$\mathrm{Tg}(x/2)$
$-1/\sin^2(ax)$	$[\mathrm{cotg}(ax)]/a$
$\sin(x)/\cos^2(x)$	$\mathrm{Sec}(x)$
$-\cos(x)/\sin^2(x)$	$\mathrm{Cosec}(x)$
$\sin^2(ax)$	$½\,x - [\sin(2ax)]/[4a]$
$\cos^2(ax)$	$½\,x + [\sin(2ax)]/[4a]$
$\sin^3(x)$	$(1/12)[9\sin(x)+\sin(3x)]$
$\cos^3(x)$	$(1/12)[-9\cos(x)+\cos(3x)]$
$\arcsin(ax)$	$x\arcsin(ax) + (1/a)(\sqrt{1-a^2x^2})$
$\arccos(ax)$	$x\arccos(ax) - (1/a)(\sqrt{1-a^2x^2})$
$\mathrm{arctg}(ax)$	$x\,\mathrm{arctg}(ax) - (\ln(1+a^2x^2))/(2a)$
$\mathrm{arccot}(ax)$	$x\,\mathrm{arccot}(ax) + (\ln(1+a^2x^2))/(2a)$
$1/\sqrt{(a^2-x^2)}$	$\mathrm{Arcsin}(x/a)$
$-1/\sqrt{(a^2-x^2)}$	$\mathrm{Arccos}(x/a)$
$1/(a^2+x^2)$	$(1/a)\,\mathrm{arctg}(x/a)$
$-1/(a^2+x^2)$	$(1/a)\,\mathrm{arccotg}(x/a)$
$1/(x\sqrt{(x^2-a^2)})$	$(1/a)\arccos(a/x)$
$-1/(x\sqrt{(x^2-a^2)})$	$(1/a)\arcsin(a/x)$
$1/(x\sqrt{(x^2\pm a^2)})$	$(1/a)\ln\lvert x / [a+\sqrt{(a^2\pm x^2)}]\rvert$
$\sqrt{(a^2-x^2)}$	$½\,[a^2\arcsin(x/a) + x\sqrt{(a^2-x^2)}$
$\sqrt{(x^2\pm a^2)}$	$(x/a^2)\sqrt{(x^2\pm a^2)} \pm (a^2/2)\ln\lvert x+\sqrt{(x^2\pm a^2)}\rvert$
$x/\sqrt{(x^2+a)}$	$\sqrt{(x^2+a)}$
$x/\sqrt{(a-x^2)}$	$-\sqrt{(a-x^2)}$
$1/\sqrt{(x^2\pm a^2)}$	$\mathrm{Ln}\lvert x+\sqrt{(x^2\pm a^2)}\rvert$
$1/(x^2-a^2)$	$(1/2a)\ln\lvert(x-a)/(x+a)\rvert$
$1/(a^2-x^2)$	$(1/2a)\ln\lvert(a+x)/(a-x)\rvert$
$1/\cos(ax) = \sec(ax)$	$(\ln\lvert\tan(½\,ax + \pi/4)\rvert)/a$
$1/\sin(ax) = \mathrm{cosec}(ax)$	$(\ln\lvert\mathrm{tg}(½\,ax)\rvert)/a$
$\mathrm{cotg}(ax)$	$(\ln\lvert\sin(x)\rvert)/a$
$\mathrm{tg}(ax)$	$-(\ln\lvert\cos(ax)\rvert)/a = (\ln\lvert\sec(ax)\rvert)/a$
$e^{\alpha x}\cos(\beta x)$	$[e^{\alpha x}/(\alpha^2+\beta^2)][\alpha\cos(\beta x)+\beta\sin(\beta x)]$
$e^{\alpha x}\sin(\beta x)$	$[e^{\alpha x}/(\alpha^2+\beta^2)][\alpha\sin(\beta x)-\beta\cos(\beta x)]$

Sostituzioni per integrali trigonometrici:

$t = \mathrm{tg}(x/2)$; $\cos(x) = (1-t^2)/(1+t^t)$; $\sin(x) = 2t/(1+t^2)$;
$x = 2\,\mathrm{arctg}(t)$; $dx = [2/(1+t^2)]dt$

Sostituzione generale delle variabili in un integrale triplo

$$\begin{cases} x = \alpha(u,t,w) \\ y = \beta(u,t,w) \\ z = \gamma(u,t,w) \end{cases} \qquad \lvert I\rvert = \begin{vmatrix} \partial x/\partial u & \partial x/\partial t & \partial x/\partial w \\ \partial y/\partial u & \partial y/\partial t & \partial y/\partial w \\ \partial z/\partial u & \partial z/\partial t & \partial z/\partial w \end{vmatrix}$$

$\iiint_V f(x,y,z)\,dxdydz = \iiint_{V'} f(\alpha,\beta,\gamma)\,\lvert I\rvert\,du\,dt\,dw$

Coordinate Polari:

$x = \rho\cos(\theta)$; $y = \rho\sin(\theta)$; $I = \rho$

Coordinate Cilindriche:

$x = \rho\cos(\theta)$; $y = \rho\sin(\theta)$; $z = z$; $I = \rho$

Coordinate sferiche:

$x = r\sin(\varphi)\cos(\theta)$; $y = r\sin(\varphi)\sin(\theta)$; $z = r\cos(\varphi)$
$I = r^2\sin(\varphi)$

Integrale Curvilineo:

$\int_{(M),(N)} X(x,y)dx + Y(x,y)dy =$
$= \int_{\alpha,\beta} \{ X[\varphi(t),\psi(t)]\,\varphi'(t) + Y[\varphi(t),\psi(t)]\,\psi'(t) \}\,dt$

Formula di Gauss - Green (Area)

$\iint_D [(\partial X/\partial y) - (\partial Y/\partial x)]\,dxdy = \int_{L(orario)} X\,dx + Y\,dy$

Integrale di Superfice o Flusso di F attraverso σ

$\iint_\sigma \mathbf{F}\,\mathbf{n}\,d\sigma = \iint_\sigma [X\cos(n,x) + Y\cos(n,y) + Z\cos(n,z)]\,d\sigma =$
$= \iint_\sigma X\,dydz + Y\,dzdx + Z\,dxdy$

Formula di Stokes (Int. Superficie → Int. Curvilineo)

$\int_l \mathbf{F}\,ds = \iint_\sigma \mathbf{n}\,\mathrm{rot}(\mathbf{F})\,d\sigma$
$\int_\lambda X dx + Y dy + Z\,dz = \iint_\sigma [\,(\partial Z/\partial y)-(\partial Y/\partial z))\cos(n,x) +$
$+ ((\partial X/\partial z)-(\partial Z/\partial x))\cos(n,y) +$
$+ ((\partial Y/\partial x)-(\partial X/\partial y))\cos(n,z)]\,d\sigma$

Formula di Ostrogradskij - Gauss (Volume → Superficie)

$\iiint_V \mathrm{div}(\mathbf{F})dv = \iint_\sigma \mathbf{F}\,\mathbf{n}\,ds$
$\iiint_V [(\partial X/\partial x) + (\partial Y/\partial y) + (\partial Z/\partial z)]\,dx\,dy\,dz =$
$= \iint_\sigma (\,X\cos(n,x) + Y\cos(n,y) + Z\cos(n,z)\,)\,d\sigma$

Area di superficie $z = f(x,y)$

$\sigma = \iint_D \sqrt{[\,1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2]}\,dx\,dy$

Area di una superfice di rotazione $[ds^2=dr^2+dy^2]$

$\sigma = 2\pi\int r\,dl = 2\pi\int r\sqrt{[1 + (dr/dy)^2]}\,dy$

Volume di un solido di rotazione $V = \pi\int r^2\,dy$

Baricentro

$x_G = [\iint_D \sigma\,x\,dx\,dy]/[\iint_D \sigma\,dx\,dy]$; $y_G = [\iint_D \sigma\,y\,dx\,dy]/[\iint_D \sigma\,dx\,dy]$

Momento d'Inerzia

$I = \iint_D \sigma\,\delta^2\,dx\,dy$; δ: distanza dall'asse di rotazione; σ: densità

Momento Statico $M = \iint_D \sigma\,\delta\,dx\,dy$; δ: distanza; σ: densità

Operatore Hamiltoniano (Nabla)

$\nabla = (\partial/\partial x)\mathbf{i} + (\partial/\partial y)\mathbf{j} + (\partial/\partial z)\mathbf{k}$

Gradiente: ∇f

Divergenza: $\nabla\cdot\mathbf{u}$

Rotore: $\nabla\wedge\mathbf{u}$

Proprietà:

$\nabla f_1 f_2 = f_1\nabla f_2 + f_2\nabla f_1$
$\nabla\cdot f\mathbf{u} = f\nabla\cdot\mathbf{u} + \mathbf{u}\cdot\nabla f$
$\nabla\wedge f\mathbf{u} = f\nabla\wedge\mathbf{u} + \nabla f\wedge\mathbf{u}$
div **rot u** =0; **rot grad** f =0;

Limiti, Serie, Numeri Complessi

Limiti notevoli:

$\lim_{x\to 0} [(1-\cos^{\alpha}(x))/ x^{\beta}] = \begin{cases} 0 \Leftarrow \beta=1 \\ \alpha/2 \Leftarrow \beta=2 \\ \infty \Leftarrow \beta>2 \end{cases}$ $\lim_{x\to 0}[(\alpha/\beta)(x/x^{(\beta-1)})] =$

$\lim_{x\to 0} [tg^{\alpha}(x)/x^{\beta}] =$ $\lim_{x\to 0} [1/x^{(\beta-\alpha)}] = \begin{cases} 1 \Leftarrow \beta=\alpha \\ \infty \Leftarrow \beta>\alpha \\ 0 \Leftarrow \beta<\alpha \end{cases}$

$\lim_{x\to 0} [sen^{\alpha}(x)/x^{\beta}] =$ $\lim_{x\to 0} [1/x^{(\beta-\alpha)}] = \begin{cases} 1 \Leftarrow \beta=\alpha \\ \infty \Leftarrow \beta>\alpha \\ 0 \Leftarrow \beta<\alpha \end{cases}$

$\lim_{x\to 0} [(a^{(\alpha x)}-1)/x^{\beta}] =$ $\lim_{x\to 0}[(\alpha\ln(a))/x^{(\beta-1)}] = \begin{cases} 1 \Leftarrow \beta=\alpha \\ \infty \Leftarrow \beta>\alpha \\ 0 \Leftarrow \beta<\alpha \end{cases}$

$\lim_{x\to 0} [\log_a(1+x)/x] = \log_a(e)$

$\lim_{x\to 0} [(1+x)^{1/x}] = e$

$\lim_{x\to +\infty} [(1+(b/x))^x] = e^b$

$\lim_{x\to +\infty} [(1-(1/x))^x] = 1/e$

$\lim_{x\to \infty} [\sqrt[n]{(n^{\alpha})}] = 1$

$\lim_{x\to +\infty} [\sqrt[n]{(n!)}] = +\infty$

$\lim_{x\to +\infty} [\sqrt[n]{(n!/n^n)}] = 1/e$

Forme indeterminate:

$+\infty-\infty$; $0\cdot\infty$; $0/0$; ∞/∞; 0^0; 1^{∞}; ∞^0;
$\log_0(0)$; $\log_{\infty}(0)$; $\log_1(1)$; $\log_{\infty}(\infty)$

Serie di Taylor:

$f(x) = \sum_{n,\,0,\,\infty} [\, f^{(n)}(x_0)\cdot(x-x_0)^n / n!\,]$

Serie di Fourier:

$f(t) = a_0/2 + \sum_{n,\,1,\,\infty}[a_n\cos(n\omega_0 t)+b_n\sin(n\omega_0 t)]$

$\omega_0 = 2\pi/T = \pi/l$; $T=2l$; $a_0 = (1/l) \int_{-l,l} f(t)\, dt$;

$a_n = (1/l) \int_{-l,l} f(t) \cos(n\omega_0 t)\, dt$;

$b_n = (1/l) \int_{-l,l} f(t) \sin(n\omega_0 t)\, dt$;

per f(t) pari :

$a_n = (2/l) \int_{0,l} f(t)\cos(n\omega_0 t)\, dt$; $b_n = 0$;

per f(t) dispari :

$b_n = (2/l) \int_{0,l} f(t)\sin(n\omega_0 t)\, dt$; $a_n = 0$;

Forma esponenziale:

$f(t) = \sum_{n,-\infty,\infty} [C_n\, e^{in\omega t}]$;

$C_n = (1/2l) \int_{-l,l} f(t)\, e^{-in\omega t}\, dt$

Serie di Laurent:

$f(z) = \sum_{n,-\infty,\infty} [C_n\, (z-z_0)^n]$

$C_n = (1/2\, \pi i) \int_{\sigma} f(z)/(z-z_0)^{n+1}\, dz$

Componente caratteristica o singolare:

$S_1 = \sum_{n,-\infty,-1} [C_n\, (z-z_0)^n]$

Componente olomorfa:

$S_2 = \sum_{n,0,\infty} [C_n\, (z-z_0)^n]$

Residuo:

$\Re(f) = C_{-1} = (1/2\pi i) \int_{\sigma} f(z)\, dz$

$\Re(z_{\infty}) = -C_{-1}$ (residuo all'infinito)

Per un polo di ordine 1:

$\Re(z_0) = \lim_{z\to z_0} [(z-z_0)\, f(z)]$

Per un polo di ordine n:

$\Re(z_0) = (1/(n-1)!) \lim_{z\to z_0} (d/dz)^{(n-1)}[(z-z_0)^n\, f(z)]$

Data $f(z) = A(z)/B(z)$

con $B(z_0)=0$; $A(z_0)\neq 0$; $B'(z_0) \neq 0$

si ha: $\Re_{z0}[f(z)] = A(z_0)/B'(z_0)$

Unità immaginaria: $i = \sqrt{[-1]}$

proprietà: $\sqrt{[-x]} = i\sqrt{[x]}$; $i^2 = -1$;

$i^{4k} = 1$; $i^{4k+1} = i$; $i^{4k+2} = -1$; $i^{4k+3} = -i$

Posto:

Modulo: $\rho = \sqrt{[a^2+b^2]}$; $\rho > 0$

Argomento: $\theta = arctg(b/a)$; $0 \leq \theta \leq 2\pi$

$a = \rho \cos(\theta)$

$b = \rho \sin(\theta)$

Numero Complesso in notazione :
algebrica, trigonometrica, esponenziale

$z = a + ib = \rho(\cos(\theta) + i \sin(\theta)) = \rho e^{i\theta}$

Complesso Coniugato

$z^* = a - ib = \rho(\cos(\theta) - i \sin(\theta)) = \rho e^{-i\theta}$

Operazioni

$(a+ib)+(c+id)=(a+c)+i(b+d)$

$(a+ib)-(c+id)=(a-c)+i(b-d)$

$(a+ib)\cdot(c+id)=(ac-bd)+i(ad+bc)$

$= \rho_1\cdot\rho_2[\cos(\theta+\phi)+ i\ sen(\theta+\phi)]$

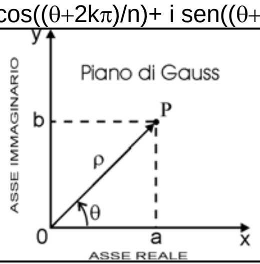
$(a+ib)/(c+id)=((ac+bd)+i(bc-ad))/(c^2+d^2)$

$= (\rho_1/\rho_2)[\cos(\theta-\phi)+ i\ sen(\theta-\phi)]$

$z^n = \rho^n[\cos(n\theta)+ i\ sen(n\theta)] = \rho^n e^{in\theta}$

$\sqrt[n]{[z]} = \sqrt[n]{[\rho]}\cdot e^{i((\theta+2k\pi)/n)} =$

$= \sqrt[n]{[\rho]}\cdot[\cos((\theta+2k\pi)/n)+ i\ sen((\theta+2k\pi)/n)]$



Trasformate

Trasformata di Fourier:

$F[f;\omega] = F[\omega] = \int_{-\infty,\infty} f(\xi)\, e^{-i\omega\xi}\, d\xi$

per f(t) pari (trasformata coseno):

$F_c[\omega] = \int_{0,\infty} f(\xi) \cos(\omega\xi)\, d\xi$

per f(t) dispari (trasformata seno):

$F_s[\omega] = \int_{0,\infty} f(\xi)\ sen(\omega\xi)\, d\xi$

Antitrasformata di Fourier:

$F^{-1}[F[\omega];t] = f(t) = (1/2\pi) \int_{-\infty,\infty} e^{i\omega t}\, F(\omega)\, d\omega$

per f(t) pari (antitrasformata coseno):

$f(t) = (2/\pi) \int_{0,\infty} F_c[\omega] \cos(\omega t)\, d\omega$

per f(t) dispari (antitrasformata seno):

$f(t) = (2/\pi) \int_{0,\infty} F_s[\omega]\ sen(\omega t)\, d\omega$

Densità spettrale:

$|F[\omega]| = |\int_{-\infty,\infty} f(\xi)\, e^{-i\omega\xi}\, d\xi|$

Proprietà della trasformata di Fourier:

- Linearità:

$F[\alpha_1 f_1+ \alpha_2 f_2 ; \omega] = \alpha_1 F[f_1;\omega] + \alpha_2 F[f_2;\omega]$

- Traslazione:

$F[f(t-h) ; \omega] = e^{-i\omega h}\, F[f(t) ; \omega]$

$F[e^{iht} f(t) ; \omega] = F[f(t) ; \omega-h]$

- Convoluzione:

$F[f(t)\circledast g(t) ; \omega] = F[f(t) ; \omega]\cdot F[g(t) ; \omega]$

$F[f(t)\cdot g(t)] = (1/2\pi)(F[f(t)]\circledast F[g(t)])$

con $f(t)\circledast g(t) = \int_{-\infty,\infty} f(\tau)g(t-\tau)\, d\tau$

- Derivazione:

$F[f^{(m)} ; \omega] = (i\omega)^m\, F[f ; \omega]$

$(d/d\omega)^m\, F[\omega] = F[(-it)^m f(t) ; \omega]$

Trasformata di Laplace:

$L[f;p] = \int_{0,\infty} e^{-pt} f(t)\, dt$

Antitrasformata di Laplace (Riemann - Fourier):

$L^{-1}[L(f)] = f(t) = (1/2\pi i) \int_{(k-i\infty),(k+i\infty)} e^{pt}\, L[f]\, dp$

Proprietà della trasformata di Laplace:

- Linearità:

$L[\alpha_1 f_1+ \alpha_2 f_2] = \alpha_1 L(f_1) + \alpha_2 L(f_2)$

- Traslazione:

$L[f(t) ; p-k] = L[e^{kt} f(t)]$

$L[f(t-a)\cdot H(t-a)] = e^{-ap}\, L[f(t)]$

- Derivazione (formule fondamentali):

$(d/dp)^k\, L[f(t)] = (-1)^k\, L[t^k f(t)]$

$L[f^{(n)}(t)] = p^n\, L[f(t)] - p^{n-1} f(0) - p^{n-2} f'(0) - ... - f^{(n-1)}(0)$

- Convoluzione (teorema di Borel):

$L[f(t)\circledast g(t)] = L[f(t)]\cdot L[g(t)]$

con $f(t)\circledast g(t) = \int_{0,t} f(\tau)g(t-\tau)\, d\tau$

- Integrale dell'originale:

$L[\int_{0,t} f(t)\, dt] = (1/p)\, L[f(t)]$

- Regola di similitudine:

$L[f(at)] = (1/a)\, L[f; p/a]$

Formula di Heavside:

$N(p)/Q(p) = \Sigma_{k,1,n} (N(\alpha_k)/Q'(\alpha_k))/(p-\alpha_k)$

con N(p) polinomio di grado m

Q(p) polinomio di grado n>m e radici α_k semplici

$L^{-1}[N(p)/Q(p)] = \Sigma_{k,1,n} (N(\alpha_k)/Q'(\alpha_k))\, e^{\alpha_k t}$

Alfabeto Greco:

Α – α	alfa
Β – β	beta
Γ – γ	gamma
Δ – δ	delta
Ε – ε	epsilon
Ζ – ζ	zeta
Η – η	eta, ita
Θ – θ	theta, thita
Ι – ι	iota
Κ – κ	cappa
Λ – λ	lamda
Μ – μ	mu
Ν – ν	nu
Ξ – ξ	csi
Ο – ο	omicron
Π – π	pi
Ρ – ρ	ro
Σ – σ	sigma
Τ – τ	tau
Υ – υ	upsilon
Φ – φ	fi
Χ – χ	chi
Ψ – ψ	psi
Ω – ω	omega

Equazioni Differenziali Ordinarie (ODE)

1° Ordine:

- **A variabili separate:**

$M(x)dx + N(y)dy = 0$ Si integra semplicemente.

- **A variabili separabili:**

$dy/dx = M(x)N(y) \Rightarrow dy/N(y) = M(x)dx$

- **Riducibili ad Equazioni omogenee:**

$dy/dx = (ax+by+c)/(a_1x+b_1y+c_1)$

posto $x = x_1+h$; $y = y_1+h \Rightarrow dy/dx = (ax_1+by_1)/(a_1x_1+b_1y_1)$

con $ah+bk+c = 0$; $a_1h+b_1k+c_1 = 0$

se $\begin{vmatrix} a & b \\ a_1 & b_1 \end{vmatrix} = 0$ allora $a_1 = \lambda a$; $b_1 = \lambda b$

posto $z = ax + b \Rightarrow$

$dy/dx = (1/b)(dz/dx) - (a/b) = (z+c)/(\lambda_z +c_1)$

- **Lineari del primo ordine:**

$(dy/dx) + P(x)y = Q(x) \Rightarrow y(x) = e^{-\int P(x)dx} [C + \int Q(x)\, e^{\int P(x)dx}]$

- **Di Bernoulli:**

$dy/dx + P(x)y = Q(x)y^n$ Si divide per y^n e si pone $z = y^{1-n}$

$dz/dx + (1-n)P(x)z = (1-n)Q(x)$

- **A differenziali totali:**

$M(x,y)\, dx + N(x,y)\, dy = 0$ se $\partial M/\partial y = \partial N/\partial x$ allora

$\int M(x,y)\, dx + \int N(x,y)\, dy - \int(\partial/\partial y)[\int M(x,y)\, dx]\, dy = C$

- **Di Clairaut:**

$y = (dy/dx)x + \psi(dy/dx)$ posto $p = dy/dx$ si ha:

$y = xp + \psi(p)$ derivando su x

$p = (dp/dx)x + p + (dp/dx)\, \psi'(p) \Rightarrow (dp/dx)[x + \psi'(p)] = 0$

da cui:

$dp/dx = 0 \Rightarrow p = C \Rightarrow y = Cx + \psi(C)$ soluzione generale

$x + \psi'(p) = 0 \Rightarrow y = xP(x) + \psi(P(x))$ soluzione particolare

- **Di Lagrange:**

$y = x\Phi(y') + \psi(y')$ posto $p = dy/dx \Rightarrow y = x\Phi(p)+ \psi(p)$

si deriva su x: $p - \Phi(p) = [x\Phi'(p)+ \psi'(p)](dp/dx)$

se p = cost $\Rightarrow p - \Phi(p) = 0 \Rightarrow y = x\Phi(p_0)+ \psi(p_0)$

altrimenti $dx/dp - x[\Phi'(p)/(p- \Phi(p)] = \psi'(p)/ (p- \Phi(p)$

da cui $x = \omega(p,C)$

eliminando p si ottiene l'integrale generale.

2° Ordine:

- **Omogenee a coefficienti costanti:**

$y'' + p\, y' + q\, y = 0$

posto $k^2 + pk +q = 0$ si trovano le radici k_1 e k_2

si hanno 3 casi :

1. k_1 e k_2 reali distinti: $k_1 \neq k_2$

$y = C_1y_2 + C_2\, y_2 = C_1\, e^{k1x} + C_2\, e^{k2x}$

2. k_1 e k_2 complessi coniugati: $k_1 = \alpha + i\beta$; $k_2 = \alpha - i\beta$

$y = e^{\alpha x} (C_1 \cos\beta x + C_2 \text{ sen } \beta x)$

3. k_1 e k_2 reali coincidenti: $k_1 = k_2$

$y = e^{k1x} (C_1 + C_2\, x)$

- **Non omogenee:**

$y'' + p\, y' + qy = f(x) \Rightarrow y = y_G+ y_P$

dove:

y_G = soluzione generale dell'eq. omogenea $y''+py'+qy = 0$

y_P = integrale particolare.

⇩

Metodo della variazione delle costanti:

$y_G = C_1\, y_1 + C_2\, y_2$ soluzione dell'eq. omogenea

si risolve il sistema:

$$\begin{cases} C'_1\, y_1 + C'_2\, y_2 = 0 \\ C'_1\, y'_1 + C'_2\, y'_2 = f(x) \end{cases}$$

da cui: $C_1 = \int C'_1\, dx + C_3$; $C_2 = \int C'_2\, dx + C_4$

$y_P = C_1\, y_1 + C_2\, y_2$